From the angle A cut *any* triangle XAY equal to the given area. Construct the triangle ZAZ' where Z and Z' lie on the sides of the angle A, and such that $AZ=AZ'=\sqrt{(AX.AY)}$. Then the area of this triangle is also equal to the given area. All lines cutting from the angle A a triangle having this area are tangent to an hyperbola having A as center and the sides of the angle as asymptotes. ZZ' is the tangent at a vertex. The circle with A as center and tangent to ZZ' is the auxiliary circle. It will cut the sides of the angle A in points which we will call M, N. At these points erect perpendiculars to the sides of the angle; these will intersect in the corresponding focus of the hyperbola S, say. Construct the circle with PS as diameter, and if this intersects the auxiliary circle, call one point of intersection Q. The angle $\angle SQP$ is right, and hence PQ is a tangent to the hyperbola, and so cuts off from the angle A the required area. There is no solution when there is no point Q, nor, according to the limitations of the question, when the segment PQ intersects the segment BC.

We can treat the angles B and C in like manner.

Also solved by Elmer Schuyler, and A. H. Holmes.

231. Proposed by B. F. FINKEL. A. M., Drury College, Springfield. Mo.

A man starts from the vertex, A, of a right isosceles triangle ABC, right-angled at A, and walks to D, the middle point of BC; from D to E, the middle point of AC; from E to F, the middle point of AD; from F to G, the middle point of DE; from G to H, the middle point of EF; from H to I, the middle point of FG; from I to J, the middle point of HI; and so on *ad infinitum*. Find the coördinates of his limiting position. [Suggested by Dr. Crawley].

I. Solution by G. B. M. ZERR. A. M., Ph. D.. Parsons, W. Va.

From J he walks to K, the mid-point of HI. He then performs the same journeys in the triangle KIJ as in ABC, and so on. The sides of KIJ are $\frac{1}{16}$ the length of the sides of ABC. Taking A as origin, the coördinates of K are $\frac{9}{16}AD$, and $\frac{3}{16}DC=\frac{3}{16}AD$. Hence the coördinates of his limiting position are

$$\tfrac{9}{16}AD\left(1+\frac{1}{16}+\frac{1}{16^2}+\frac{1}{16^3}+\frac{1}{16^4}+\ldots\ldots\right)=\tfrac{9}{15}AD=\tfrac{3}{5}AD,$$

$$\tfrac{3}{16}AD\left(1+\frac{1}{16}+\frac{1}{16^2}+\frac{1}{16^3}+\frac{1}{16^4}+\ldots\ldots\right)=\tfrac{3}{15}AD=\tfrac{1}{5}AD.*$$

II. Solution by A. H. HOLMES, Brunswick. Maine.

Draw the lines AD, DE, EF, FG, GH, HI, IJ, JK, and KL, according to the directions of the problem. Join AG and this line will pass through the point K. This is easily seen by drawing GN parallel to DC, meeting AD in N, and KM parallel to DC, meeting AD in M, and considering the similar triangles

*Taking D as the origin, DC and DA as the axes, the coordinates of the limiting point become $\frac{1}{5}AD$, $\frac{2}{5}AD$. This result agrees thus with the one in Solution II by Mr. Holmes. ED.

AKM and *AGN*.* Similarly, a line from H to D will pass through the point L.

The point P, where AG and HD intersect is the limiting position. For every triangle within KIJ whose homologous sides are parallel to those of KIJ will have the same relation to KIJ that the latter has to ABC. Therefore the line AG passes through the right angled vertices of all such triangles within KIJ, and HD passes through the middle points of all the bases.

Let D be the origin of coördinates, DC the axis of x and DA the axis of y, then the coördinates of P are easily found to be $x=\frac{1}{5}DC$, $y=\frac{2}{5}DC$.

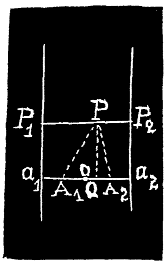
Also solved by L. E. Newcomb, Elmer Schuyler, F. D. Posey, and G. W. Greenwood.

232. Proposed by O VEBLEN, Ph. D., The University of Chicago.

Given two parallel lines a_1, a_2, and two points A_1, A_2, upon a common perpendicular to a_1, a_2 such that A_1 is at the same distance from a_1 as A_2 is from a_2. Let P_1 be the foot of the perpendicular from a point P of the same plane to the line a_1 and P_2 the foot of the perpendicular from P to a_2. Find the locus of P when $\dfrac{PA_1}{PP_1}=\dfrac{PA_2}{PP_2}$.

Solution by J. SCHEFFER, Hagerstown, Md., and A. H. HOLMES, Brunswick, Maine.

Choosing a_1a_2, a common perpendicular to the lines a_1, a_2 for the axis of x, its middle point O for the origin of orthogonal coördinates, so that $OQ=x$, $PQ=y$, and denoting $Oa_1=Oa_2$ by a, and $OA_1=OA_2$ by b, we have $PA_1=\sqrt{[y^2+(b+x)^2]}$, $PP_1=a+x$, $PA_2=\sqrt{[y^2+(b-x)^2]}$, $PP_2=a-x$.



From the condition of the problem

$$\frac{\sqrt{[y^2+(b+x)^2]}}{a+x}=\frac{\sqrt{[y^2+(b-x)^2]}}{a-x}.$$

Squaring, clearing of fractions, and simplifying, we finally and without difficulty obtain the equation

$$\frac{y^2}{b\,(a-b)}+\frac{x^2}{ab}=1.$$

If $a>b$, that is, for the case that A_1 and A_2 are situated within the parallels a_1 and a_2, the equation is that of an ellipse, whose foci are A_1 and A_2, semi-axes $\sqrt{(ab)}$, and $\sqrt{[b(a-b)}$.

If $a<b$, that is, for the case that A_1 and A_2 lie outside of the parallels a_1 and a_2, the curve is an hyperbola.

Also solved by G. B. M. Zerr, L. E. Newcomb, and G. W. Greenwood.

---

*$AN=\frac{3}{4}AD$, $NG=\frac{1}{4}AD$; $AM=\frac{9}{16}AD$, $MK=\frac{3}{16}AD$; hence $AN/NG=AM/MK$. Ed.